# On Some n-Refined Neutrosophic Groups For $3 \leq n \leq 5$

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**Abstract**

This paper is dedicated to studying some examples of n-refined neutrosophic groups and their algebraic substructures, where we deal with three different types of them, 3-refined, 4-refined, and 5-refined neutrosophic groups. Also, we present the algebraic structure of many substructures such as 3-refined neutrosophic AH-subgroups and kernels, 4-refined neutrosophic AH-homomorphisms and subgroups, and 5-refined neutrosophic AH-isomorphisms. On the other hand, many related examples will be provided to explain the algebraic concepts and their properties.

**Keywords:** 3-refined neutrosophic group; 4-refined neutrosophic group; 5-refined neutrosophic group.

## 1. Introduction

Neutrosophic algebra is the branch of neutrosophic applications that concerns the generalizations of famous classical algebraic structures into novel versions called neutrosophic algebraic structures [1-3,8-10].

In recent years, we have observed an accelerated development in the study of neutrosophic algebraic structures, since many of these structures have been studied and classified, for example, neutrosophic rings, neutrosophic matrices, neutrosophic vector spaces, and neutrosophic modules [4-7,11-13,19-22].

On the other hand, generalizations of neutrosophic algebraic structures have received a lot of attention from many researchers, as structures n-refined neutrosophic sets have been studied in many research papers. For example, n-refined neutrosophic rings, n-refined neutrosophic spaces, AH-substructures, and isometries [14-18,23-24].

n-refined neutrosophic groups have been studied in general in [25]. This has motivated us to study three different special cases, 3-refined neutrosophic groups, 4-refined neutrosophic groups, and 5-refined neutrosophic groups.

These structures will be studied in our work from many algebraic sides, especially those which are related to substructures such as AH-subgroups, AH-homomorphisms, and isomorphisms. Also, the classification of 3-refined, 4-refined, 5-refined neutrosophic groups in the case of additive abelain group will be proved and obtained.

## 2. Main results

**Definition:**

Let $(T,*)$ be a group, we define the 3-refined neutrosophic group $N_3(T)$ as follows:



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$N_3(T) = (< T \cup \{I_1, \dots, I_3\} >, *) = \{(t_0, t_1I_1, t_2I_2, t_3I_3);\ t_i \in T\}$.

The set $N_3(T)$ is closed under $*$, and it is a semi-group but not a group since $I_i$ is not invertible.

**Remark:**

If (T,+) is an additive abelian group, then the addition operation on $N_3(T)$ is defined as follows:

Consider $x = (s_0, s_1I_1, s_2I_2, s_3I_3), y = (r_0, r_1I_1, r_2I_2, r_3I_3)$ , we have

$x + y = (s_0 + r_0, [s_1 + r_1]I_1, [s_2 + r_2]I_2, [s_3 + r_3]I_3)$. In this case $(N_3(T), +)$ is an abelian group.

It can be classified as $N_3(T) \cong T \times T \times T \times T$.

**Example:**

Let $T = Z_2$ be the group of integers modulo 2, the corresponding 3-refined neutrosophic group is $N_3(T) = \{(0,0,0,0), (1,0,0,0), (0, I_1, 0,0), (0,0, I_2, 0), (1, I_1, 0,0), (1,0, I_2, 0), (0, I_1, I_2, 0), (1, I_1, I_2, 0), (1,0,0, I_1), (1,0,0, I_2), (1,0,0,1)$

$(1, I_1, I_2, 1), (1, I_1, 0,1), (1, I_2, 0,1)$ ... .... }.

**Remark:**

If T is a multiplicative group, then group product on $N_3(T)$ can be defined as follows:

For $= (s_0, s_1I_1, s_2I_2, s_3I_3), y = (r_0, r_1I_1, r_2I_2, r_3I_3)$ , we have

$xy = (l_0, l_1I_1, lI_2, l_3I_3);\ t_s = \prod_{i,j=0}^{3}(s_ir_j)I_iI_j\ ;\ I_0 = e_G\ and\ I_iI_j = I_{\min\,(i,j)}$.

The identity element is $(e_G, e_GI_1, e_GI_2, e_GI_3)$.

In this case $N_3(T)$ is not isomorphic to the direct product of 4 copies of T, since it is not a group.

**Example:**

Let $T = S_3 = \{g_0 = I, g_1, g_2, g_3, g_4, g_5\}$ be symmetric group, the 3-refined neutrosophic group is $N_3(T) = \{(m, nI_1, sI_2, kI_2); m, n, s, k \in T\}$.

**Definition:**

(a) Let $N_3(T)$ be an 3-refined neutrosophic group. It is called abelian if $x * y = y * x$ for all $x, y \in N_3(T)$.

(b) The subset $Z(N_3(T)) = \{y \in N_3(T); y * x = x * y\ for\ all\ x \in N_3(T)\}$ is called 3-refined neutrosophic center.

**Theorem:**

Let $N_3(T)$ be a 3-refined neutrosophic group. Then

(a) If G is abelian, $N_3(T)$ is abelian.

(b) $N_3(T)$is abelian if and only if $N_3(T) = Z(N_3(T))$.

**Definition:**

Let $N_3(T)$ be a 3-refined neutrosophic group, K be a nonempty subset of $N_3(T)$, we say that K is a 3-refined neutrosophic subgroup if H contains a subgroup of T.

**Definition:**

Let $N_3(T)$ be a 3-refined neutrosophic group, the number of its elements is denoted by

O($N_3(T)$).

If $N_3(T)$ is finite, then $O(N_3(T)) = k$, elsewhere $O(N_3(T)) = \infty$.

$O(N_3(T))$ is called the order of 3-refined neutrosophic group $N_3(T)$.



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**Theorem:**

Let G be a finite group, $N_3(T)$be its corresponding 3-refined neutrosophic group. Then if $O(G) = k$, we have $O(N_3(T)) = k^4$.

**Definition:**

Let $N_3(T), N_3(K)$ be two 3-refined neutrosophic groups, $h: N_3(T) \to N_3(K)$ be a well defined map, we say that $h$ is a 3-refined neutrosophic homomorphism if:

(a) $h(xy) = h(x)h(y)$ for all $x, y \in N_3(T)$.

(b) $h(e_G, e_G, \dots, I_k, e_G, \dots, e_G) = (e_G, e_G, \dots, I_k, e_G, .., e_G)$.

**Definition:**

Let $N_3(T), N_3(K)$ be two 3-refined neutrosophic groups, $h: N_3(T) \to N_3(K)$ be a 3-refined neutrosophic homomorphism, we define:

(a) $Ker(h) = \{x \in N_3(T); h(x) = e_{N_3(K)}\}$.

(b) $Im(h) = \{y \in N_3(K);\ \exists\ x \in N_3(T): h(x) = y\}$.

**Theorem:**

Let $N_3(T), N_3(K)$ be two 3-refined neutrosophic groups, $h: N_3(T) \to N_3(K)$ be a 3-refined neutrosophic homomorphism, we have:

(a) $Ker(h)$ is a 3-refined neutrosophic subgroup of $N_3(T)$.

(b) $Im(h)$ is a 3-refined neutrosophic subgroup of $N_3(K)$.

Proof:

(a) The restriction $h_G$ of $g$ is a homomorphism between

T and K, its kernel $Ker(h_G)$ will be a subset of $Ker(h)$, i.e $Ker(h)$ contains a subgroup of T, hence it is a 3-refined neutrosophic subgroup.

(b) The proof is similar to (a).

**Definition:**

Let $N_3(T), N_3(K)$ be two 3-refined neutrosophic groups, $g: N_3(T) \to N_3(K)$ be a 3--refined neutrosophic homomorphism, we call it a 3--refined neutrosophic isomorphism if it is bijective.

**Theorem:**

Let $N_3(T)$ be a 3--refined neutrosophic group. The set $TI_k = \{x * I_k; 1 \leq k \leq 3\}$ has a group structure.

Proof:

We define the following operation on $TI_k$:

$(xI_k) \times (yI_k) = (x * y)I_k$. Also, $\times$ is well defined operation, and it is associative with $I_k$ as an identity, and for each

$xI_k \in TI_k$, there is an inverse $x^{-1}I_k$; $x^{-1}$ is the inverse of $x$ in T.

**Theorem:**

Let $N_3(T)$be a 3--refined neutrosophic group, consider the previously defined group $TI_k$. Then $T \cong TI_k$.

**Definition:**

Let $T, S$ be two groups, $T \times S$ be the direct product. We define the direct product of the related 3-refined neutrosophic groups as follows:

$N_3(T) \times N_3(S) =< T \times S \cup \{I_1, .., I_3\} >$. We call it the 3-refined neutrosophic direct product.



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**Definition:**

Let $N_3(T) = \{(t_0, t_1I_1, t_2I_2, t_3I_3);\ t_i \in G\}$ be a 3-refined neutrosophic group,

$N_3(S) = \{(m_0, m_1I_1, m_2I_2, m_3I_3);\ m_i \in S_i;\ S_i\ is$ a subgroup of $T\ for\ all\ i\}$ is called an AH-subgroup of $N_3(T)$.

If $H_i \cong H_j\ for\ all\ i \neq j$, then it is called an AHS-subgroup.

**Definition:**

Let $(T,*)$ be a group, we define the 4-refined neutrosophic group $N_4(T)$ as follows:

$N_4(T) = (< T \cup \{I_1, \dots, I_4\} >,*) = \{(t_0, t_1I_1, t_2I_2, t_3I_3, t_4I_4);\ t_i \in T\}$.

The set $N_4(T)$ is closed under $*$, and it is a semi group but not a group since $I_i$ is not invertible.

**Remark:**

If (T,+) is an additive abelian group, then addition operation on$N_4(T)$ is defined as follows:

Consider $x = (s_0, s_1I_1, s_2I_2, s_3I_3, s_4I_4), y = (r_0, r_1I_1, r_2I_2, r_3I_3, r_4I_4)$ , we have

$x + y = (s_0 + r_0, [s_1 + r_1]I_1, [s_2 + r_2]I_2, [s_3 + r_3]I_3, [s_4 + r_4]I_4)$. In this case $(N_4(T), +)$ is an abelian group.

It can be classified as $N_4(T) \cong T \times T \times T \times T \times T$.

**Remark :**

If T is a multiplicative group, then group product on $N_4(T)$ can be defined as follows:

For $= (s_0, s_1I_1, s_2I_2, s_3I_3, s_4I_4), y = (r_0, r_1I_1, r_2I_2, r_3I_3, r_4I_4)$ , we have

$xy = (l_0, l_1I_1, lI_2, l_3I_3, l_4I_4);\ l_s = \prod_{i,j=0}^{4}(s_ir_j)I_iI_j\ ;\ I_0 = e_G\ and\ I_iI_j = I_{\min\,(i,j)}$.

The identity element is $(e_G, e_GI_1, e_GI_2, e_GI_3, e_GI_4)$.

In this case $N_4(T)$ is not isomorphic to the direct product of 5 copies of T, since it is not a group.

**Definition:**

(a) Let $N_4(T)$ be a 4-refined neutrosophic group. It is called abelian if $x * y = y * x$ for all $x, y \in N_4(T)$.

(b) The subset $Z\big(N_4(T)\big) = \{y \in N_4(T); y * x = x * y\ for\ all\ x \in N_4(T)\}$ is called 4-refined neutrosophic center.

**Theorem :**

Let $N_4(T)$ be a 4-refined neutrosophic group. Then

(a) If G is abelian, $N_4(T)$ is abelian.

(b) $N_4(T)$ is abelian if and only if $N_4(T) = Z(N_4(T))$.

**Definition:**

Let $N_4(T)$ be an 4-refined neutrosophic group, K be a nonempty subset of $N_4(T)$, we say that K is a 4-refined neutrosophic subgroup if K contains a subgroup of T.

**Definition:**

Let $N_4(T)$ be a 4-refined neutrosophic group, the number of its elements is denoted by

O($N_4(T)$).

If $N_4(T)$ is finite, then $O\big(N_4(T)\big) = k$, elsewhere $O\big(N_4(T)\big) = \infty$.

$O\big(N_4(T)\big)$ is called the order of 4-refined neutrosophic group $N_4(T)$.



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**Theorem:**

Let G be a finite group, $N_4(T)$be its corresponding 4-refined neutrosophic group. Then if $O(G) = k$, we have $O(N_4(T)) = k^5$.

**Definition:**

Let $N_4(T), N_4(K)$ be two 4-refined neutrosophic groups, $h: N_4(T) \to N_4(K)$ be a well defined map, we say that $h$ is a 4-refined neutrosophic homomorphism if:

(a) $h(xy) = h(x)h(y)$ for all $x, y \in N_4(T)$.

(b) $h(e_G, e_G, \dots, I_k, e_G, \dots, e_G) = (e_G, e_G, \dots, I_k, e_G, .., e_G)$.

**Definition:**

Let $N_4(T), N_4(K)$ be two 4-refined neutrosophic groups, $h: N_4(T) \to N_4(K)$ be a 4-refined neutrosophic homomorphism, we define:

(a) $Ker(h) = \{x \in N_4(T); h(x) = e_{N_4(K)}\}$.

(b) $Im(h) = \{y \in N_4(K);\ \exists\ x \in N_4(T): h(x) = y\}$.

**Theorem:**

Let $N_4(T), N_4(K)$ be two 4-refined neutrosophic groups, $h: N_4(T) \to N_4(K)$ be a 4-refined neutrosophic homomorphism, we have:

(a) $Ker(h)$ is a 4-refined neutrosophic subgroup of $N_4(T)$.

(b) $Im(h)$ is a 4-refined neutrosophic subgroup of $N_4(K)$.

Proof:

(a) The restriction $h_G$ of $g$ is a homomorphism between

T and K, its kernel $Ker(h_G)$ will be a subset of $Ker(h)$, i.e $Ker(h)$ contains a subgroup of T, hence it is a 3-refined neutrosophic subgroup.

(b) The proof is similar to (a).

**Definition :**

Let $N_4(T), N_4(K)$ be two 4-refined neutrosophic groups, $g: N_4(T) \to N_4(K)$ be a 4--refined neutrosophic homomorphism, we call it a 4--refined neutrosophic isomorphism if it is bijective.

**Theorem:**

Let $N_4(T)$ be a 4--refined neutrosophic group. The set $TI_k = \{x * I_k; 1 \le k \le 4\}$ has a group structure.

Proof:

We define the following operation on $TI_k$:

$(xI_k) \times (yI_k) = (x * y)I_k$. Also, $\times$ is well defined operation, and it is associative with $I_k$ as an identity, and for each

$xI_k \in TI_k$, there is an inverse $x^{-1}I_k$; $x^{-1}$ is the inverse of $x$ in T.

**Theorem:**

Let $N_4(T)$be a 4--refined neutrosophic group, consider the previously defined group $TI_k$. Then $T \cong TI_k$.

**Definition:**

Let $T, S$ be two groups, $T \times S$ be the direct product. We define the direct product of the related 4-refined neutrosophic groups as follows:

$N_4(T) \times N_4(S) =< T \times S \cup \{I_1, .., I_4\} >$. We call it the 4-refined neutrosophic direct product.



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**Definition:**

Let $N_4(T) = \{(t_0, t_1I_1, t_2I_2, t_3I_3, t_4I_4);\ t_i \in T\}$ be a 4-refined neutrosophic group,

$N_4(S) = \{(m_0, m_1I_1, m_2I_2, m_3I_3, m_4I_4);\ m_i \in S_i;\ S_i\ is\ a\ subgroup\ of\ T\ for\ all\ i\}$ is called an AH-subgroup of $N_4(T)$.

If $H_i \cong H_j\ for\ all\ i \neq j$, then it is called an AHS-subgroup.

**Definition:**

Let $(T,*)$ be a group, we define the 5-refined neutrosophic group $N_5(T)$ as follows:

$N_5(T) = (< T \cup \{I_1, \dots, I_5\} >,*) = \{(t_0, t_1I_1, t_2I_2, t_3I_3, t_4I_4, t_5I_5);\ t_i \in T\}$.

The set $N_5(T)$ is closed under $*$, and it is a semi-group but not a group since $I_i$ is not invertible.

**Remark:**

If (T,+) is an additive abelian group, then the addition operation on $N_5(T)$ is defined as follows:

Consider $x = (s_0, s_1I_1, s_2I_2, s_3I_3, s_4I_4, s_5I_5), y = (r_0, r_1I_1, r_2I_2, r_3I_3, r_4I_4, r_5I_5)$ , we have

$x + y = (s_0 + r_0, [s_1 + r_1]I_1, [s_2 + r_2]I_2, [s_3 + r_3]I_3, [s_4 + r_4]I_4, [s_5 + r_5]I_5)$. In this case $(N_5(T), +)$ is an abelian group.

It can be classified as $N_5(T) \cong T \times T \times T \times T \times T \times T$.

**Remark :**

If T is a multiplicative group, then the group product on $N_5(T)$ can be defined as follows:

For $= (s_0, s_1I_1, s_2I_2, s_3I_3, s_4I_4, s_5I_5), y = (r_0, r_1I_1, r_2I_2, r_3I_3, r_4I_4, r_5I_5)$ , we have

$xy = (l_0, l_1I_1, lI_2, l_3I_3, l_4I_4, l_5I_5);\ l_s = \prod_{i,j=0}^{5}(s_ir_j)I_iI_j\ ;\ I_0 = e_G\ and\ I_iI_j = I_{\min\,(i,j)}$.

The identity element is $(e_G, e_GI_1, e_GI_2, e_GI_3, e_GI_4, e_GI_5)$.

In this case $N_5(T)$ is not isomorphic to the direct product of 6 copies of T, since it is not a group.

**Definition:**

(a) Let $N_5(T)$ be a 4-refined neutrosophic group. It is called abelian if $x * y = y * x$ for all $x, y \in N_5(T)$.

(b) The subset $Z(N_5(T)) = \{y \in N_5(T); y * x = x * y\ for\ all\ x \in N_5(T)\}$ is called 5-refined neutrosophic center.

**Theorem :**

Let $N_4(T)$ be a 5-refined neutrosophic group. Then

(a) If G is abelian, $N_5(T)$ is abelian.

(b) $N_5(T)$ is abelian if and only if $N_5(T) = Z(N_5(T))$.

**Definition:**

Let $N_5(T)$ be an 5-refined neutrosophic group, K be a nonempty subset of $N_5(T)$, we say that K is a 5-refined neutrosophic subgroup if K contains a subgroup of T.

**Definition:**

Let $N_5(T)$ be a 5-refined neutrosophic group, the number of its elements is denoted by

$O(N_5(T))$.

If $N_5(T)$ is finite, then $O(N_5(T)) = k$, elsewhere $O(N_5(T)) = \infty$.

$O(N_5(T))$ is called the order of 5-refined neutrosophic group $N_5(T)$.



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**Theorem:**

Let G be a finite group, $N_5(T)$be its corresponding 5-refined neutrosophic group. Then if $O(G) = k$, we have $O(N_5(T)) = k^6$.

**Definition:**

Let $N_5(T), N_5(K)$ be two 5-refined neutrosophic groups, $h: N_5(T) \to N_5(K)$ be a well defined map, we say that $h$ is a 5-refined neutrosophic homomorphism if:

(a) $h(xy) = h(x)h(y)$ for all $x, y \in N_5(T)$.

(b) $h(e_G, e_G, \dots, I_k, e_G, \dots, e_G) = (e_G, e_G, \dots, I_k, e_G, .., e_G)$.

**Definition:**

Let $N_5(T), N_5(K)$ be two 5-refined neutrosophic groups, $h: N_5(T) \to N_5(K)$ be a 5-refined neutrosophic homomorphism, we define:

(a) $Ker(h) = \{x \in N_5(T); h(x) = e_{N_4(K)}\}$.

(b) $Im(h) = \{y \in N_5(K);\ \exists\ x \in N_5(T): h(x) = y\}$.

**Theorem:**

Let $N_5(T), N_5(K)$ be two 5-refined neutrosophic groups, $h: N_5(T) \to N_5(K)$ be a 5-refined neutrosophic homomorphism, we have:

(a) $Ker(h)$ is a 5-refined neutrosophic subgroup of $N_5(T)$.

(b) $Im(h)$ is a 5-refined neutrosophic subgroup of $N_5(K)$.

Proof:

(a) The restriction $h_G$ of $g$ is a homomorphism between

T and K, its kernel $Ker(h_G)$ will be a subset of $Ker(h)$, i.e $Ker(h)$ contains a subgroup of T, hence it is a 3-refined neutrosophic subgroup.

(b) The proof is similar to (a).

**Definition :**

Let $N_4(T), N_4(K)$ be two 4-refined neutrosophic groups, $g: N_4(T) \to N_4(K)$ be a 4--refined neutrosophic homomorphism, we call it a 4--refined neutrosophic isomorphism if it is bijective.

**Theorem:**

Let $N_5(T)$ be a 5--refined neutrosophic group. The set $TI_k = \{x * I_k; 1 \leq k \leq 5\}$ has a group structure.

Proof:

We define the following operation on $TI_k$:

$(xI_k) \times (yI_k) = (x * y)I_k$. Also, $\times$ is well defined operation, and it is associative with $I_k$ as an identity, and for each

$xI_k \in TI_k$, there is an inverse $x^{-1}I_k$; $x^{-1}$ is the inverse of $x$ in T.

**Theorem:**

Let $N_5(T)$be a 5--refined neutrosophic group, consider the previously defined group $TI_k$. Then $T \cong TI_k$.

**Definition:**

Let $T, S$ be two groups, $T \times S$ be the direct product. We define the direct product of the related 5-refined neutrosophic groups as follows:

$N_5(T) \times N_5(S) =< T \times S \cup \{I_1, .., I_5\} >$. We call it the 5-refined neutrosophic direct product.



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**Definition:**

Let $N_5(T) = \{(t_0, t_1I_1, t_2I_2, t_3I_3, t_4I_4, t_5I_5);\ t_i \in T\}$ be a 5-refined neutrosophic group,

$N_5(S) = \{(m_0, m_1I_1, m_2I_2, m_3I_3, m_4I_4, m_5I_5);\ m_i \in S_i;\ S_i\ is\ a\ subgroup\ of\ T\ for\ all\ i\}$ is called an AH-subgroup of $N_5(T)$.

If $H_i \cong H_j\ for\ all\ i \neq j$, then it is called an AHS-subgroup.

## 3. Conclusion

In this paper, we have defined the concept of 3,4,5-refined neutrosophic groups. Also, we have introduced some of their substructures such as 3-refined neutrosophic subgroup/homomorphism, 4-refined neutrosophic homomorphism/subgroup, and 5-refined neutrosophic isomorphism/homomorphism. Many examples were produced to explain the abstract algebraic concepts and theorems.

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